ON AN APPROXIMATE METHOD OF SOLUTION OF NONLINEAR HEAT-CONDUCTION PROBLEMS

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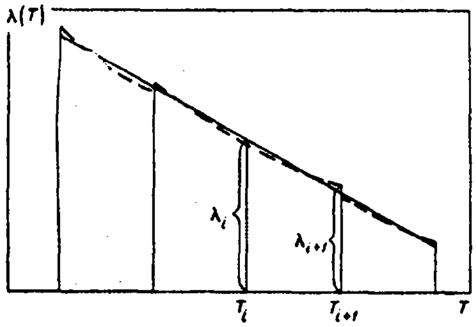
An approximate method is presented for the solution of problems of transient heat conduction in solids with thermal conductivity and specific heat linearly dependent on temperature.

Consider the process of transient heat conduction in a solid whose thermophysical properties are given



Approximation of the thermal conductivity λ(T) by exponential curves. Continuous line—exact relation; Broken line—approximation.

functions of temperature. The problem can be reduced to the nonlinear heat-conduction equation

$$\rho(T)C(T)\frac{\partial T}{\partial\tau}=\mathrm{div}[\lambda(T)\,\mathrm{grad}\,T] \qquad (1)$$

with the appropriate initial and boundary conditions. Charnyi [1] has presented an approximate method of solution for this equation for the case when ρC = const and the thermal conductivity λ is a linear function of temperature. In this work we shall consider the case when both λ and the specific heat C vary with temperature.

It is known that the density of most solid materials can be taken as constant and that the variation of λ and C with temperature follows the linear laws

$$\lambda(T)=\lambda_0+nT, \qquad (2)$$

$$C(T)=C_0+mT. \qquad (3)$$

These functions can be approximated within the temperature range under consideration, or within a portion of it if the whole range is too large, by the exponential functions

$$\lambda(T)=\lambda_0+nT=\lambda_i\exp\left(\frac{T-T_i}{T_{i+1}-T_i}\ln\frac{\lambda_{i+1}}{\lambda_i}\right), \qquad (4)$$

$$C(T)=C_0+mT=C_i\exp\left(\frac{T-T_i}{T_{i+1}-T_i}\ln\frac{C_{i+1}}{C_i}\right). \qquad (5)$$

Here λ_i, λ_{i+1}, C_i, and C_{i+1} are the approximate values of λ and C at the ends of the chosen temperature interval $\Delta T = T_{i+1} - T_i$ (Figure). These values are chosen so as to yield equal areas under the original and the approximate curves. Thus

$$\lambda_{i+1}-\lambda_i=\left[\lambda_0+\frac{n}{2}(T_i+T_{i+1})\right]\ln\frac{\lambda_{i+1}}{\lambda_i},$$

$$C_{i+1}-C_i=\left[C_0+\frac{m}{2}(T_i+T_{i+1})\right]\ln\frac{C_{i+1}}{C_i}.$$

The approximate values of λ(T) and C(T) at the ends of ΔT should be chosen so that the exponential curves will be sufficiently close to the exact linear curves.

Using C, and then λ, as new independent variables, we can linearize equation (1):

$$\rho C(T)\frac{dT}{dC}\frac{\partial C}{\partial\tau}=\mathrm{div}\left[\lambda\left(\frac{C}{m}-\frac{C_0}{m}\right)\frac{dT}{dC}\,\mathrm{grad}\,C\right],$$

$$\rho\frac{m(T_{i+1}-T_i)}{\ln(C_{i+1}/C_i)}\frac{dC}{d\lambda}\frac{\partial\lambda}{\partial\tau}=\mathrm{div}\left(\lambda\frac{dC}{d\lambda}\,\mathrm{grad}\,\lambda\right), \qquad (6)$$

$$\rho\frac{m\ln(\lambda_{i+1}/\lambda_i)}{n\ln(C_{i+1}/C_i)}\frac{\partial\lambda}{\partial\tau}=\nabla^2\lambda.$$

The boundary conditions for equation (1),

$$T_{\tau=0}=T_{in},\quad T_s=f(\tau); \qquad (7)$$

$$\left.\begin{aligned}&\text{or}\quad -\lambda(T)(\mathrm{grad}\,T)_s=g(\tau),\\ &\text{or}\quad -\lambda(T)(\mathrm{grad}\,T)_s=\alpha(T_s-T_a),\end{aligned}\right\} \qquad (8)$$

Table 1

Thermophysical Properties of the Cube

$t_{i+1}-t_i$	λ_i	λ_{i+1}	C_i	C_{i+1}
250—0	11.80	9.57	923.189	1021.579
500—250	9.83	7.44	1030.790	1124.993
750—500	7.44	5.70	1124.993	1239.293
1000—750	5.27	3.82	1222.546	1356.523

yield the boundary conditions for the variable λ:

$$\lambda_{\tau=0}=\lambda_0+nT_{in}, \qquad (9)$$

$$\left.\begin{aligned}&\lambda_s=\lambda_0+nf(\tau),\\ &\text{or}\quad -\frac{T_{i+1}-T_i}{\ln(\lambda_{i+1}/\lambda_i)}(\mathrm{grad}\,\lambda)_s=g(\tau),\\ &\text{or}\quad -\frac{T_{i+1}-T_i}{\ln(\lambda_{i+1}/\lambda_i)}(\mathrm{grad}\,\lambda)_s=\alpha\left(\frac{\lambda_s}{n}-\frac{\lambda_0}{n}-T_a\right).\end{aligned}\right| \qquad (10)$$

Table 2

Results Obtained by the Present Method (p. m.) Compared with the Data of [2]

τ, sec	t (0; 0; 0; τ), °C			t (0.1; 0; 0; τ), °C			t (0; 0.1; 0.1; τ), °C			t (0.1; 0.1; 0.1; τ), °C		
	p.m.	[z] [2]	δ, %	p.m.	[z] [2]	δ, %	p.m.	[z] [2]	δ, %	p.m.	[z] [2]	δ, %
0	1000	1000	0.0	1000	1000	0.0	1000	1000	0.0	1000	1000	0.0
792	995	982	1.3	885	843	5.0	763	709	7.6	628	580	8.3
1584	900	891	1.0	675	645	4.7	487	454	7.3	365	315	1.6
2376	721	703	2.6	445	430	3.4	312	275	1.3	186	179	3.9
3168	430	449	4.4	273	247	1.0	162	164	1.2	115	108	6.5
3960	223	230	3.1	144	143	0.7	100	96	4.2	71	63	13

Thus, using the relations (4) and (5), we can reduce the nonlinear heat-conduction problem (1), (7), (8) to the linear system (6), (9), (10). For example, consider the cooling of a cube [2] with edges of length $l = 0.4$ m. The data are: $t_{in} = 1000°C$, $t_s = 0°C$, $\lambda = 11.63 - 0.00814t$ W/(m · degC), $C = 921.096 + 0.419t$ J/(kg · degC), $\rho = 3000$ kg/m^3.

Table 1 shows the values of $\Delta t = t_{i+1} - t_i$, λ_{i+1}, λ_i, C_{i+1}, C_i.

Table 2 compares the values of temperature $t(x, y, z, \tau)$, obtained by the present method with those given in [2].

REFERENCES

1. I. A. Charnyi, Izv. AN SSSR, Otdelenie tekhnicheskikh nauk, no. 6, 1951.

2. M. A. Mikheev, Fundamentals of Heat Transfer [in Russian], Gosenergoizdat, 1956.

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